Total No. of Questions : 8] [Total No. of Printed Pages : 4

(1126)

B.A./B.Sc. (General) Ist Semester (0001) Examination

0045

MATHEMATICS

Paper : III

(Trigonometry and Matrices)

Time : 3 Hours] [Maximum Marks : 30

Note :- Attempt *five* questions in all by selecting at least *two* questions from each Unit.

Unit–I

1. (a) If α, β are roots of $t^2 - 2t + 2 = 0$, then prove that

$$\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \frac{\sin n\phi}{\sin^n \phi}$$

where $x + 1 = \cot \phi$

(b) Apply De Moivre's theorem to find an equation whose roots are the n^{th} power of roots of equation

$$x^2 - 2x \cos \theta + 1 = 0$$ 3,3

A-20

P.T.O.

2. (a) Solve the equation :

$$x^4 + x^3 + x^2 + x + 1 = 0$$

(b) Express $\cos^8 \theta$ in terms of cosines or sines of multiple of θ.

3. (a) Prove that :

$$\frac{\pi}{4} = \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{8}\right) - \frac{1}{3}\left(\frac{1}{2^3} + \frac{1}{5^3} + \frac{1}{8^3}\right) + \frac{1}{5}\left(\frac{1}{2^5} + \frac{1}{5^5} + \frac{1}{8^5}\right) + \ldots\ldots$$

(b) Show that the sum of $(n + 1)$ terms of the series $\sin \alpha + n \sin (\alpha + \beta) + \frac{n(n-1)}{2} \sin (\alpha + 2\beta) + \ldots\ldots$ is

$$2^n \cos^n\left(\frac{\beta}{2}\right) \cos\left(\alpha + \frac{n\beta}{2}\right)$$

4. (a) If $\log [\sin (\theta + i\phi)] = \alpha + i\beta$, show that :

$$\cosh 2\phi - \cos 2\theta = 2e^{2\alpha}$$

(b) For any integer m, find

$$\sum_{k=1}^{n} \left[exp\left(\frac{2\pi ik}{n}\right)\right]^m .$$

Unit–II

5. (a) Prove that B'AB is symmetric or skew-symmetric according as A is symmetric or skew-symmetric.

(b) Express :

$$A = \begin{pmatrix} 1+i & 2i & 3 \\ 0 & 2-3i & 3-4i \\ 5 & -7i & 0 \end{pmatrix}$$

as the sum of a Hermitian and skew-Hermitian matrix.

(c) Is $\left(\overline{A}\right)^1 = \overline{(A^1)}$? Justify your answer. 2,2,2

6. (a) For what value of y the matrix :

$$P = \begin{pmatrix} y+a & b & c \\ a & y+b & c \\ a & b & y+c \end{pmatrix}$$

has rank 3 ?

(b) Find rank of :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$$

after reduction to normal form. 3,3

7. (a) Investigate for what values of λ and μ, the equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

has (i) no solution (ii) unique solution (iii) infinite solutions.

(b) Check for linear dependence or linear independence the following rectors : 3

$u = (2, 3, 1)$, $v = (-1, 4, 2)$, $w = (1, 18, -4)$

8. (a) Show that the metrix

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

is diagonalizable over the set of complex number system $\mathbb{C}$. Also find an invertible matrix P over $\mathbb{C}$ such that p^{-1} AP is a diagonal matrix.

(b) State and prove Cayley-Hamilton theorem. 3

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